Universidad de los Andes

DEPARTAMENTO DE FÍSICA

INTRODUCTION TO CELESTIAL MECHANICS

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NOMBRE DEL CURSO: Introduction to Celestial Mechanics
CÓDIGO DEL CURSO: FISI XXXX
UNIDAD ACADÉMICA: Departamento de Física
PERIODO ACADÉMICO: 2025-I
HORARIO:
SALÓN:
CRÉDITOS: 3 (pregrado), 4 (posgrado)
REQUISITOS: Física I, Vector calculus
(Necessary mathematics will be covered during the course so that this course can be taken by physicists and engineers)

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HORARIO Y LUGAR DE ATENCIÓN: Ip403

I Introduction

How do the heavenly bodies move and what is the relation between their movements? What natural forces govern such movements? Celestial Mechanics (CM) is the discipline that attempts to answer such questions and studies the motion of planets, their satellites, asteroids, comets as well as artificial satellites and space probes. It traces its origin to ancient civilizations and has followed the development of humankind from antiquity to the space exploration age. For tens of thousands of years, humans studied the heavens with just the naked eye and tried to follow the motion of the fascinating luminous objects in the sky. We are now equipped with advanced technology and a better understanding of the fundamental physics and mathematics behind the movement of the heavenly objects. This progress has led to interesting discoveries and questions related to our solar system and outside.

The subject of computational CM goes back to the times of Johannes Kepler and is as such old. However, due to recent discoveries such as those of exoplanets, the migration of Jupiter in our solar system and the connection to General Relativity, there is a renewed interest in the subject. Some facts to understand the above in more detail: Mercury, Venus, Earth and Mars are known collectively as the rocky planets, in contrast to the Solar System's gas giants - Jupiter, Saturn, Uranus and Neptune. Rocky planets including the Earth, have a composition that is about one-third metallic iron and two-thirds rock. Mercury is the other way around. The "grand tack hypothesis"proposes that the inner Solar System was sculpted by the giant planets' orbital migration in the gaseous protoplanetary disk. Jupiter formed at a distance of 3.5 AU from the Sun, then migrated inward before reversing course due to capturing Saturn in an orbital resonance and later ending up in its current orbit at 5.2 AU. Similar migrations have however not been confirmed in exoplanetary systems. Studies of such phenomena are mostly carried out within the realm of classical mechanics and Newton's theory of gravitation though exceptional circumstances and very precise measurements sometimes necessitate the use of Einstein's general theory of relativity. A well known example is the anomalous rate of the precession of Mercury's orbit.

The methodology of Celestial Mechanics has evolved from its original form in the 18^{th} and 19^{th} centuries along with the development of new techniques of high precision observations, computer generation, development of spatial dynamics, and progress in mathematics and theoretical physics. Chaos is also an important player since the solutions of equations of CM show great sensitivity to initial conditions. One last point to keep in mind is that present-day CM is not restricted to gravitational forces but rather to many other forces such as gas drag, thermal emissions, interactions between radiation and matter, to name a few that perturb the motion of natural and artificial celestial bodies in a significant way.

II Description of the course

The course begins with some historical and elementary introduction to the basic concepts in mechanics and celestial mechanics followed by a treatment of the two body problem. We shall then go over to understand the movement of celestial bodies through Kepler's laws and the computational methods for studying the orbits. Some aspects of the Newton's theory of gravitation will then be addressed in connection with astrophysical objects. After a brief look into the concept of rotating reference frames, we shall try to understand the restricted three body problem. This is one of the most interesting problems of celestial mechanics. Imagine a system of three masses where two are comparable and much larger than a third one. This could be the case of a planet with two suns. The course ends with presentations by students on topics they have chosen for the term papers or a short lecture.

III Objectives

The main objectives of the course are:

- to provide the students with an understanding of the motion of celestial bodies
- to gain knowledge on the key methods of celestial mechanics and its astrophysical applications.

IV Skills developed

On completion of the course, the student is expected:

- to develop a background on the basic concepts in mechanics as applied to celestial mechanics
- to become familiar with the two body and restricted three body problem of celestial mechanics
- to gain the knowledge on the methods for the computation of orbits of celestial bodies
- to develop the competence in applying theoretical knowledge in practice through the exercises and homeworks given during the course
- to get experience in working as part of a team and getting started on the methods of finding and reporting information in literature (see Term Paper below)

V Weekly syllabus

The following topics will be covered during the semester (16 weeks).

- Week 1: Celestial mechanics: definition and brief historical introduction of it's development
  The Celestial sphere: altazimuth coordinates (ac) and equatorial coordinates (ec)
  Conversion between (ac) and (ec)
  Ecliptic coordinates
  The mean Sun, precession of the equinoxes, nutation and the length of the year

- Week 2: Revision of Newtonian mechanics, kinematics and dynamics of curvilinear motion (we will use these concepts again in week 5)

- Week 3: Two body problem:
  Motion of the centre of mass
  Relative motion and reduction to the one body problem
  The integral of areas (orientation of the orbital plane in space)
  Laplace-Runge-Lenz vector

- Weeks 4 and 5: Keplerian Orbits:
  Kepler's laws: first and third laws from Newton's laws, second law from conservation of angular momentum
  Orbital parameters and orbital energies
  Different orbits and positions in orbits: transfer, elliptical, planetary, parabolic, hyperbolic
  Binary star systems

- Week 6: Computation of orbits (I):
  Elements of an orbit for general trajectories using position and velocity vectors
  Elliptical orbit: Gauss's method

- Week 7: Computation of orbits (II):
  Euler's theorem
  Parabolic orbit: Method of Olbers
  Rectilinear orbit
  Orbit of an extrasolar planet

- Week 8: Gravitational potential (GP):
  GP and potential energy
  GP generated by an axially symmetric mass distribution
  GP due to a uniform sphere
  GP outside a uniform spheroid
  GP due to a uniform ring
  An example: rocky asteroid

- Weeks 9 and 10: Motion of celestial bodies in a central potential:
  motion of an object in a general central force field
  Stability of circular orbits in a central force field
  Perihelion precession of planets and apsidal angle
  Perihelion precession of Mercury

- Week 11: Rotating reference frames:
  Centrifugal acceleration, Coriolis force
  Rotational flattening
  Tidal elongation, tidal torques
  Roche radius

- Week 12 - 13: Restricted three body problem (I):
  Equations of motion with respect to inertial and rotating systems
  Jacobi constants
  Regions of zero velocity
  Tisserand's relation

- Week 14: Restricted three body problem (II):
  Particular solutions: Lagrange's points
  Realistic bodies en Lagrange's points and stability near the Lagrange's points

- Week 15 and 16: Presentations

VI Methodology

Since the present course is a theoretical course, the methodology involves teaching on the white board with a regular interaction with students. As mentioned below, the students will choose a topic to write a term paper and hence it will sometimes involve outside the classroom discussions with the students on topics related to the course.

VII Grades and evaluation criteria

There will be two partial exams during the semester. The undergraduate students also have to present a term paper. The term paper consists of an article written by the student on a topic closely connected with the syllabus of this course. The student is expected to choose a topic and then find relevant information in books and literature and some research articles (can also be from journals with pedagogic articles) which discuss the issue. Having chosen the topic, the student can discuss the topic outside lecture hours with the lecturer if he/she finds it necessary. Based on this, he/she should be able to make a write-up on the topic and present a short talk on the same to fellow students.
This will be done as a team of about 3 to 4 students.

Partial exams (for all students): (30 %) and (20 %)

Term Paper (undergraduate students): 20 % (to be submitted at the end of week 16),
Presentation of term paper (undergraduate students): 5 %

Short lecture (postgraduate students): Prepare a lecture of 40 minutes on a special topic in celestial mechanics not covered in the course and present it to the students (20 %)
Prepare a set of 3 exercises on the special topic lecture (5 %)

Final Exam (for all students): 25 %

Homeworks will be given at regular intervals but will not be graded. Hints and solutions to do them will be discussed in class.

VIII Bibliography

The topics given in the syllabus above can be found in the following books.
Main references:

1 *An introduction to Celestial Mechanics*, Richard Fitzpatrick, Cambridge University Press (2012).

2 *Principios de mecánica celeste*, Jose Gregorio Portilla Barbosa, Universidad Nacional de Colombia (2018).

Additional references:

3 *Celestial Mechanics*, J. B. Tatum (2023), available at https://www.astro.uvic.ca/ tatum/celmechs.html

4 *Foundations of Celestial Mechanics*, Elena Bannikova and Massimo Capaccioli, Springer (2022).

5 *The Sheer Joy of Celestial Mechanics*, Nathaniel Grossman, Birkhäuser (1996).

6 *The Geometry of Celestial Mechanics*, Hansjörg Geiges, Cambridge University press (2016).

7 *Celestial Mechanics: The Waltz of the Planets*, A. Celletti and E. Perozzi, Springer-Praxis (2007).

8 *Methods of Celestial Mechanics*, Gerhard Beutler, Springer-Verlag (2005).

9 *Mathematical Introduction to Celestial Mechanics*, Harry Pollard, Printice-Hall (1966).